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Clean time-dependent string backgrounds from bubble baths

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Abstract: We consider the set of controlled time-dependent backgrounds of general relativity and string theory describing "bubbles of nothing", obtained via double analytic continuation of black hole solutions. We analyze their quantum stability, uncover some novel features of their dynamics, identify their causal structure and observables, and compute their particle production spectrum. We present a general relation between squeezed states, such as those arising in cosmological particle creation, and nonlocal theories on the string worldsheet. The bubble backgrounds have various aspects in common with de Sitter space, Rindler space, and moving mirror systems, but constitute controlled solutions of general relativity and string theory with no external forces. They provide a useful theoretical laboratory for studying issues of observables in systems with cosmological horizons, particle creation, and time-dependent string perturbation theory.

Keywords: Superstrings and Heterotic Strings, M-Theory, Supersymmetry Breaking, Cosmology of Theories beyond the SM.

Contents

1. Introduction

Many static backgrounds are known in which string/M theory is understood relatively well and can be studied very concretely; some of these backgrounds are even semi-realistic at low energies. In particular, tori, orbifolds, Calabi-Yau manifolds, and anti-de Sitter (AdS) spacetimes provide very useful and explicit backgrounds which have taught us much about the phenomena and formalism in the theory.

Our knowledge of time-dependent backgrounds in quantum gravity is far more rudimentary. We do not have a formulation of string perturbation theory which applies to generic time-dependent backgrounds, and we do not have a clear notion of what the observables are in a general background (there being in general no guarantee of the existence of an S-matrix) [1, 2]. Obviously, understanding string theory in time-dependent backgrounds is necessary for applying string theory to cosmology, and may be useful for relating string theory to nature. In particular, one very basic process we would like to get a handle on is that of particle creation.

In order to attack these questions, we need good examples of time-dependent solutions that are under complete theoretical control. Finding such backgrounds is more difficult than in the static case. Time-dependent backgrounds rich enough to exhibit particle creation are nonsupersymmetric at low energies, since time translation (and thus the set of transformations generated by supercharges that commute to time translation) is not a symmetry.[1] A generic homogeneous cosmology evolves from a singularity, at which point the classical spacetime description breaks down, or evolves toward a singularity in the future. Aside from de Sitter space,[2] the standard examples used for studying the behavior of quantum field theory in time-dependent curved spacetimes [5] are either singular (such as cosmological and black hole geometries), involve external forces put in by hand (such as in Rindler space or moving mirror examples), or simply are not solutions to the Einstein equations even at long distances. String/M theory may ultimately resolve the singularities in the first set of cases, but at present we do not even know how to formulate string theory in any of these cases. It seems reasonable to start by considering nonsingular time-dependent backgrounds. Although general theorems [6] guarantee a large class of such solutions, e.g., describing the nonlinear scattering of gravitational waves, essentially none are known explicitly.

In this paper we study a relatively simple set of time-dependent non-singular solutions to general relativity and string/M theory which exhibit a number of interesting phenomena. We start with the observation [7] that double analytical continuations of rotating, uncharged black hole solutions [8], which have been much studied as part of destabilizing instanton processes following [9], constitute a set of interesting time-dependent backgrounds in their own right, which can be stabilized against quantum mechanically induced tadpoles. These spacetimes are smooth and geodesically complete, and they look like "bubbles of nothing" in an asymptotically flat space, which contract until they reach a finite (tunably large in string units) radius, at which point they start to expand. These geometries have aspects in common with various features of de Sitter space, Rindler space, and moving mirror configurations, but they constitute weakly curved solutions of general relativity and string/M theory (which can be arbitrarily weakly coupled in the string theory case).

These solutions have a compact (periodic) dimension, with fermions obeying non-supersymmetric boundary conditions as they go around it, and we find an interesting connection between the dynamics of the bubble and the asymptotic behavior of the compact dimension (far from the bubble). When the compact dimension remains finite, the bubbles continue to expand exponentially indefinitely. However, when the compact dimension opens up (and the asymptotic space looks locally like Minkowski space), the bubbles slow down and stop accelerating. This connection is found for the solutions in all dimensions. When these solutions are embedded in supergravity or string theory, this is equivalent to saying that the bubbles expand exponentially into regions with broken supersymmetry, but ultimately stop accelerating in directions in which there is asymptotic local supersymmetry (namely, the supersymmetry breaking effects go to zero asymptotically).

[1] It would be interesting to try to exploit the spatial version of supersymmetry introduced in [3] to try to see if that provides any protection from quantum instabilities.

[2] A noncritical string construction for de Sitter space is under investigation [4].

For the analytic continuation of the Schwarzschild black hole [9],[3] the size of the compact dimension remains finite asymptotically, and the bubble continues to expand exponentially. We will show that this solution has cosmological horizons (similar to de Sitter space) of infinite area (in contrast to de Sitter space). We will argue that this spacetime is classically stable (at least in four dimensions). Quantum mechanically, it is known to be unstable [11]. So, although it provides an interesting example of a stable classical solution to string theory with cosmological horizons,[4] it is ultimately not a suitable background.

On the other hand, for analytic continuations of even dimensional Kerr black holes with all rotation parameters nonzero, the compact dimension opens up in all directions. The geometry near the bubble begins (and ends) in a phase similar to a Milne universe, with spatial directions near the bubble contracting (expanding) linearly in proper time, and the region far from the bubble reducing to flat Minkowski space. In between these periods of mild contraction and expansion, for a tunably long period, the bubble geometry contracts and then expands exponentially. This solution has no horizons and we will argue that perturbative quantum corrections do not destabilize it (assuming it is stable classically). The S-matrix in these backgrounds is well defined.

Perhaps of most interest are the intermediate cases of bubbles obtained from even dimensional Kerr black holes which rotate in some directions but not in others (obviously, this requires at least six dimensions). These are hybrid examples in which the bubble accelerates eternally in some directions, but stops in other directions. In particular, bubbles in orbifolds of the "twisted circle" form $[(\mathbb{C}^q \times S^1)/\mathbb{Z}_N] \times \mathbb{R}^{8-2q,1}$ (see [12]) experience accelerated expansion along the $\mathbb{R}^{8-2q,1}$ directions, but remain near the origin of the complex planes in the $(\mathbb{C}^q \times S^1)/\mathbb{Z}_N$ directions. We will argue that these backgrounds are also quantum mechanically stable perturbatively, yet have horizons for certain observers. These horizons are somewhat unusual since although two such observers lose causal contact at late times, they can both send signals to a third observer located in a different direction. These hybrid examples are unstable to nonperturbative quantum processes corresponding to nucleation of additional bubbles. This presumably does not occur in the case of bubbles with all rotation parameters nonzero, or at least, it would be very unlikely.

In addition to understanding the basic observables such as the S-matrix, one would like to understand how to perform perturbative field theory and string theory computations in these backgrounds. One basic phenomenon that arises in generic time-dependent backgrounds is particle creation. In quantum field theory in curved spacetime, this involves the possibility of different choices of vacuum for the fields. One basic question is how this ambiguity arises in string theory. In section 5 we show that in general, calculating matrix elements between squeezed states, such as those corresponding to baths of particles generated by cosmological particle creation, corresponds to working with a Nonlocal String Theory [13] on the string worldsheet. The bubble backgrounds provide a textbook example of particle creation, with early and late epochs of mild time-dependence (and Minkowski

[3]For some further discussion of various bubble solutions see [10].

[4]The curvature can be made arbitrarily small, in which case the α' corrections should only shift the solution negligibly given the absence of tachyonic or marginal modes.

JHEP07(2002)007

null infinity) interrupted by a long epoch of stronger time-dependence. We compute particle creation in field theory for modes of wavelength shorter than the size of the bubble, and translate this to the leading order in α' nonlocal action on the string worldsheet. We also comment on a number of basic issues involved in formulating string perturbation theory in time-dependent backgrounds.

Although we are able to address the evident quantum mechanically induced instabilities in these (non-supersymmetric) backgrounds, we are not able to rule out the existence of classical tachyonic instabilities arising from fluctuations of the metric in the most general cases. This is related to the fact that due to the complication of the coupled linear perturbation equations, the classical stability of Kerr black holes in higher than four dimensions has not yet been established. However, we do rule out classical tachyons from modes of scalar fields and from metric fluctuations in the case of the four-dimensional Schwarzschild bubble. It would be interesting to investigate this issue further.

It would also be interesting to investigate D-branes in our backgrounds, and to analyze double analytic continuations of more general black holes, such as asymptotically AdS or dS black holes.

Recently, a number of other interesting time-dependent backgrounds have been introduced and studied, for example [14, 15, 4], [16]–[24]. The backgrounds we study here are complimentary to these in many ways; in particular the particle creation per mode in our backgrounds is finite as opposed to the situation in [16].

A brief outline of this paper is the following. In the next section we study the bubbles obtained by analytic continuation of the Schwarzschild black hole. We discuss their dynamics, horizons, classical stability, and quantum instability. In section 3, we investigate the Kerr bubbles, starting with the simplest four dimensional case. We then move on to the higher dimensional bubbles, discussing both those coming from fully rotating and partially rotating black holes. Finally, we discuss the quantum stability of these solutions. In section 4 we give a quantum field theory calculation of particle creation in the four dimensional Kerr bubble. The final section contains a discussion of particle creation in string theory, and of general issues associated with defining string theory in these bubble backgrounds and more general time-dependent backgrounds.

2. Schwarzschild bubbles

In this section we study spacetimes obtained from double analytic continuation of Schwarzschild black holes [9]. We will show explicitly that these vacuum solutions have horizons analogous to de Sitter spacetime (as was suggested independently by Petr Hořava). We also argue that they are classically stable. However, they do not represent good backgrounds for string theory since there is a nonzero quantum correction to the stress energy tensor asymptotically (this is essentially a Casimir energy) which destabilizes the spacetime. Nevertheless, these solutions are a convenient starting point since they are simpler and illustrate some of the features we will see in the stable examples discussed in the next section.

JHEP07(2002)007

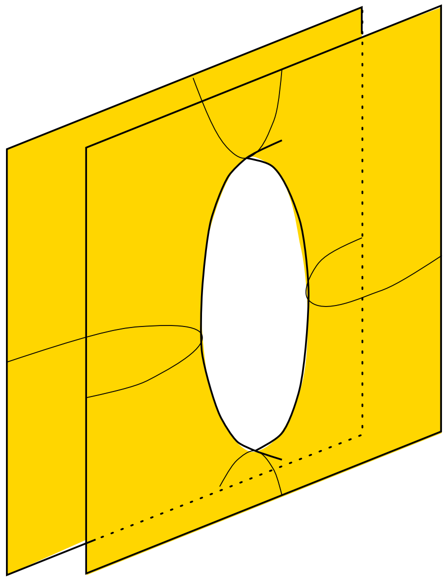

Figure 1: A schematic depiction of the geometry at a fixed time, with the χ circle replaced by two points, and the r and ϕ directions manifest (drawing courtesy of Petr Hořava).

2.1 Classical solutions

Consider a Schwarzschild black hole in D spacetime dimensions, with metric

$$ds^2 = -\left[1-\left(\frac{r_0}{r}\right)^{D-3}\right]dt^2 + \left[1-\left(\frac{r_0}{r}\right)^{D-3}\right]^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\Omega_{D-3}^2\right), \qquad (2.1)$$

where $d\Omega_{D-3}^2$ is the metric on a unit S^{D-3}. One can obtain a lorentzian vacuum solution to Einstein's equations [25, 9] by the following double analytic continuation:

$$t \equiv i\chi\,, \qquad \theta - \frac{\pi}{2} \equiv i\tau\,. \qquad (2.2)$$

The metric becomes

$$ds^2 = \left[1-\left(\frac{r_0}{r}\right)^{D-3}\right]d\chi^2 + \left[1-\left(\frac{r_0}{r}\right)^{D-3}\right]^{-1}dr^2 + r^2(-d\tau^2 + \cosh^2\tau\; d\Omega_{D-3}^2)\,. \qquad (2.3)$$

The radial variable is now restricted to the range $r \geq r_0$, and regularity at $r = r_0$ requires that χ be periodic with period $4\pi r_0/(D-3)$ (as in the standard euclidean black hole background, where we only make the first continuation in (2.2)). Thus, the spacetime asymptotically has one direction compactified on a circle. The solution is invariant under the large symmetry group: $\mathrm{U}(1)\times\mathrm{SO}(D-2,1)$. The maximum curvature is of order $1/r_0^2$, which can be made arbitrarily small by taking r_0 large. There are no singularities and the spacetime is geodesically complete.

This solution describes a contracting and then expanding "bubble of nothing" in the following sense. Consider the geometry on the $\tau = 0$ surface. This resembles $\mathbb{R}^{D-2}$ with a sphere of radius r_0 removed. Over each point is a circle whose radius smoothly goes to zero at $r = r_0$ (see figure 1). Thus $r = r_0$ is not a boundary of the space, but it is the S^{D-3} of minimal area. As we move away from the $\tau = 0$ surface (both to the future and past) the area of this minimal sphere grows exponentially. We will call this minimal surface, given by $r = r_0$, the "bubble". The event horizon of the original black hole becomes the bubble after the analytic continuations. Note that the geometry traced out by the bubble is precisely de Sitter space. In effect, this solution embeds de Sitter space into an asymptotically flat vacuum solution. Even though the bubble appears to be undergoing constant acceleration, the curves at $r = r_0$ (and constant point on S^{D-3}) are geodesics. This follows from the fact that $r = r_0$ is the fixed point of the $\partial/\partial\chi$ symmetry. If these curves had a nonzero acceleration, it would be a preferred vector orthogonal to $\partial/\partial\tau$ at $r = r_0$, and there are no such preferred vectors.

2.2 Horizons

We now describe the causal structure of the Schwarzschild bubble (2.3). We will show that this spacetime has (observer dependent) horizons analogous to de Sitter spacetime. More precisely, we show that any two observers which stay at different points on S^{D-3} lose causal contact at late time (they do not have to stay on the bubble $r = r_0$). It clearly suffices to consider null curves from one observer to the other which move in the (r, χ, φ) directions where φ is a parameter along the geodesic in S^{D-3} connecting the position of the two observers. Given such a curve $x^\mu(\lambda)$ with tangent vector $\xi^\mu = \dot{x}^\mu$, the condition that it be null, $\xi^\mu \xi_\mu = 0$, is

$$\dot{\chi}^2 f(r) + \frac{\dot{r}^2}{f(r)} - \dot{\tau}^2 r^2 + \dot{\varphi}^2 r^2 \cosh^2\tau = 0\,, \tag{2.4}$$

where

$$f(r) \equiv 1 - \left(\frac{r_0}{r}\right)^{D-3}\,. \tag{2.5}$$

It follows that

$$\dot{\varphi}^2 \cosh^2\tau \leq \dot{\tau}^2\,. \tag{2.6}$$

If the null curve starts at some late time τ_0, we have $|\dot{\varphi}| \leq e^{-\tau}\dot{\tau}$, so the maximum change in φ to the future is $|\Delta\varphi_{\max}| = e^{-\tau_0}$. This shows that any two observers that stay at different φ will lose causal contact at late times.

This result is easy to understand intuitively when both observers are sitting on the bubble $r = r_0$. We know that de Sitter space has horizons, so two observers that stay at different points on the S^{D-3} cannot communicate by null curves that stay on the bubble. But a null curve that moves in the r, χ plane projects onto a timelike curve in the de Sitter space, and hence has even less chance of being used to send a signal between observers at different points on the sphere. Of course it would be very strange if one could send a signal by a null curve but not by a null geodesic. This does not happen. Within the de Sitter space at $r = r_0$, the boundary of the region that can communicate with an event p consists of (de Sitter) null geodesics from p. One can show that these geodesics are also null geodesics of the full bubble spacetime. In general, null geodesics which start at constant r and χ, stay at constant r and χ.

JHEP07(2002)007

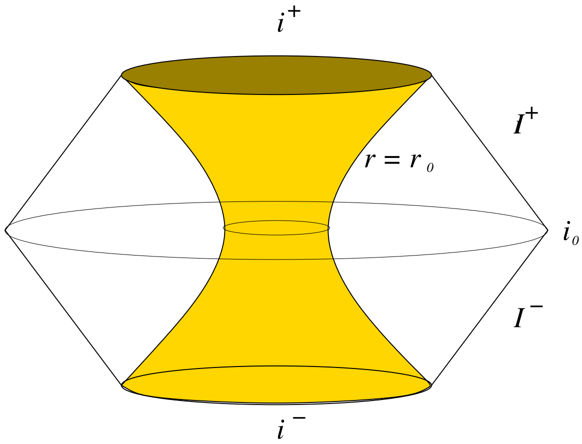

Figure 2: Penrose diagram of Schwarzschild bubble. The space inside the shaded region is absent; its surface at $r = r_0$ is the bubble which traces out de Sitter space.

We now show that the only restriction on whether observers can communicate at late time is the one we have just discussed: that they have the same coordinates on the S^{D-3}. Let ξ^μ now be tangent to a null geodesic moving in the r, χ plane, but not moving on the S^{D-3}. There are two conserved quantities $P_\chi = \dot{\chi} f(r)$ and $E = r^2 \dot{\tau}$.[5] So the condition $\xi^\mu \xi_\mu = 0$ now yields

$$\dot{r}^2 - \frac{E^2}{r^2} f(r) = -P_\chi^2 \,. \tag{2.7}$$

The second term is an effective potential which vanishes both at $r = r_0$ and at $r = \infty$. So, typical null geodesics oscillate between a maximum and minimum value of r. Clearly, by changing E and P_χ, one can find null geodesics which connect observers at any two values of r. Similarly, from the definition of P_χ we see that $|\dot{\chi}| \geq |P_\chi|/f(r_{max})$. So, as long as $P_\chi \neq 0$, the null geodesic goes around the χ circle infinitely many times and can easily connect observers at different χ.

What is the Penrose diagram of the Schwarzschild bubble? This is a little subtle. In drawing Penrose diagrams, one usually suppresses the spheres of spherical symmetry. However, if we do this, we will not see the horizons, since they only exist for observers at different points on the sphere. So we will keep one direction on the sphere. Another problem is that due to the Kaluza-Klein boundary conditions, one cannot conformally rescale the metric and add a smooth boundary at null infinity. We will avoid this by suppressing the χ direction. The remaining spacetime is shown in figure 2.

It is conformally equivalent to the region $X^\mu X_\mu \geq 1$ in Minkowski spacetime. One can think of this as a spacetime in which future and past timelike infinity, i^+, i^-, are no longer points, but spread out into spacelike circles (or spheres in the full spacetime). It is easy to verify that timelike geodesics also oscillate between maximum and minimum values of r, so all timelike geodesics have endpoints on i^- and i^+.

[5] Even though $\partial/\partial\tau$ is not a symmetry of (2.3), it is a symmetry of the three dimensional spacetime obtained by fixing a point on S^{D-3}.

The Schwarzschild bubble has no event horizons in the black hole sense: the past of future null infinity includes the entire spacetime. This is clear since every event can send a light ray to at least some points on $\mathcal{I}^+$, e.g. the generator with the same position on S^{D-3}. (But unlike Minkowski spacetime, the light rays from an event p do not reach an entire cross-section of $\mathcal{I}^+$. Some of the null generators of $\mathcal{I}^+$ cannot receive signals from p.) However, as we have seen, there are cosmological horizons analogous to de Sitter space in the sense that the past of every complete timelike geodesic is not the entire spacetime. As in de Sitter space, if we consider the vacuum determined by analytic continuation of the path integral on the euclidean black hole background, static observers on the bubble would see a thermal bath with a temperature of order $1/r_0$ (so they may consider themselves to be in a bubble bath). One important difference from de Sitter space concerns the horizon area. If we define the area of the horizon by considering the boundary of the past of the endless geodesic (as one does in asymptotically de Sitter spacetimes), it is clear that the horizon area is infinite. So, there does not seem to be a finite dimensional Hilbert space associated with these spacetimes, and the issue raised in [26] does not arise.

2.3 Classical stability

The metric (2.3) is a classical solution to Einstein's equations. Let us now investigate its stability, beginning at the classical level. This requires checking whether there are normalizable effectively tachyonic modes (namely, modes which grow in time relative to the background metric) among the solutions to the linearized field equations. Such modes would be localized near the bubble since the asymptotic region has no such excitations.

We begin by considering a scalar field ϕ. In this background, it satisfies the following equation of motion (where we set $r_0 = 1$ for simplicity, it can always be restored by dimensional analysis) :

$$\frac{1}{(1-\frac{1}{r^{D-3}})}\partial_\chi^2\phi + \frac{1}{r^{D-2}}\partial_r\left[r^{D-2}\left(1-\frac{1}{r^{D-3}}\right)\partial_r\phi\right] + \frac{1}{r^2}\nabla^2_{dS_{D-2}}\phi = m^2\phi\,, \tag{2.8}$$

where m^2 is the bare mass. Let us separate variables and consider modes with definite de Sitter mass M^2, $\nabla^2_{dS_{D-2}}\phi = M^2\phi$, and definite momentum k around the χ direction, $\partial_\chi^2\phi = -k^2\phi$. For now let us just consider modes with $k = 0$ and particles with zero mass, $m = 0$.

The solutions at large and small $(r-1)$ are as follows. At large r, we have solutions

$$\phi \sim r^{-\frac{(D-3)}{2}\pm i\mu}, \tag{2.9}$$

where as in [2], $\mu = \sqrt{M^2 - \frac{(D-3)^2}{4}}$. The solution which goes as $\phi \sim r^{-(D-3)/2-|\mu|}$ (for $M^2 < \frac{(D-3)^2}{4}$) becomes more and more normalizable as M^2 becomes more negative, and it is normalizable for $M^2 < \frac{(D-3)^2}{4} - 1$.

For r near the bubble, by changing coordinates to $r = 1 + \rho^2$, we find a series solution

$$\phi = \phi_0\left(1 - \frac{M^2}{(D-3)}\rho^2 + \dots\right) = \phi_0\left(1 - \frac{M^2}{(D-3)}(r-1) + \dots\right), \tag{2.10}$$

and another singular solution which diverges logarithmically. We want to know if there is any value of M^2 such that the smooth series solution (2.10) matches onto the normalizable solutions $\phi \sim r^{-(D-3)/2-|\mu|}$, with $M^2 < \frac{(D-3)^2}{4} - 1$. In fact, it is easy to see that the full solution is monotonic and thus (2.10) cannot match onto the decaying solution at large r, for any $M^2 < 0$. Let us take without loss of generality $\phi(1) = \phi_0 = 1$. Our equation of motion (2.8) is (again for $m = k = 0$)

$$\frac{1}{r^{D-2}}\partial_r\left[r^{D-2}\left(1-\frac{1}{r^{D-3}}\right)\partial_r\phi\right] = -\frac{1}{r^2}M^2\phi\,, \tag{2.11}$$

which can be written in the form

$$\frac{1}{r^{D-2}}\partial_r[a(r)\partial_r\phi] = \frac{1}{r^{D-2}}(a'\phi' + a\phi'') = -\frac{1}{r^2}M^2\phi, \tag{2.12}$$

where $a > 0$ and $a' > 0$ everywhere. Now, from (2.10) we know that for $M^2 < 0$, ϕ' starts out positive, so the mode begins growing from its starting value of $\phi_0 = 1$ at the bubble. The question is then whether it can turn around, reaching some maximum positive value of ϕ, $\phi_c \equiv \phi(r_c)$ for some r_c. This would require $\phi'(r_c) = 0$. The equation (2.12) would then be at r_c

$$\frac{1}{r^{D-2}}\partial_r[a(r)\partial_r\phi]\bigg|_{r=r_c} = \frac{1}{r^{D-2}}(a\phi'')\bigg|_{r=r_c} = -\frac{1}{r_c^2}M^2\phi. \tag{2.13}$$

But, since the right-hand side of this is greater than zero for negative M^2, so is the left-hand side. Since $r > 0$ and $a > 0$ always, this would mean we would have to have $\phi'' > 0$, a contradiction. So there can be no point where the mode turns over, and the large r continuation of (2.10) must always include the nonnormalizable solution at large r. By a simple rescaling of ϕ, this argument generalizes to arbitrary m ($m^2 > 0$) and k, and to positive M^2 with $M^2 < (D-3)^2/4 - 1$.

It also appears to generalize, given the metric perturbations of the $D = 4$ Schwarzschild black hole studied explicitly in [27, 28], to metric perturbations in $D = 4$, at least for $k = 0$ (higher k modes might be expected to be less tachyonic in any case). One can translate the modes studied in [27, 28], expressed in terms of tensor spherical harmonics in the black hole background, to modes in the bubble geometry. In the former case, the spherical harmonics are eigenfunctions of the laplacian on the spherical directions of the black hole with eigenvalue $-l(l+1)$; in our analytic continuation these become tensor spherical harmonics on the de Sitter slices of the Schwarzschild bubble geometry with $M_{dS}^2 = -l(l+1)$. The time direction in the black hole becomes our χ direction, and imaginary frequency for the black hole modes (corresponding to tachyons in that geometry) would correspond to real momentum k along our χ circle, which for us is quantized. The analysis of [27, 28] rules out tachyons in the black hole background by showing that there are no normalizable and nonsingular solutions to the radial part of the equations of motion, and this directly rules out tachyonic modes in the bubble of $M_{dS}^2 = -l(l+1)$. Furthermore, we find that by replacing $-l(l+1)$ by M^2 in their analysis, and rescaling fields appropriately, one can extend their arguments to arbitrary M^2, at least for $k = 0$. Thus, at least in four dimensions, the Schwarzschild bubble appears to be a classically stable solution to the equations of motion.

In $D > 4$ dimensions, there are two possibilities. One can consider the D dimensional bubble solution (2.3), or one can take a product of a lower dimensional bubble and another Ricci-flat solution such as flat space. In the latter case there is a possible instability analogous to the Gregory-Laflamme instability of black strings [29]. This arises since the euclidean Schwarzschild metric has a negative mode [30]. In other words, there is a transverse, traceless $h_{\mu\nu}$ satisfying $\triangle_L h_{\mu\nu} = -\lambda^2 h_{\mu\nu}$, where λ^2 is of order $1/r_0^2$ and $\triangle_L$ is the Lichnerowicz operator obtained from the linearized Einstein equation. This mode is spherically symmetric and independent of χ. Thus, it can be analytically continued to a real perturbation on either the Schwarzschild black hole or the bubble spacetime. Of course, by itself, it does not define a physical perturbation of the lorentzian spacetime since it does not satisfy the linearized vacuum field equations ($\triangle_L h_{\mu\nu} = 0$). But if there are extra flat directions with coordinates x^i, one can consider $h_{\mu\nu}e^{iq_i x^i}$ with $q^2 = \lambda^2$. For the black string, this is a static perturbation. For the bubble, it is invariant under the de Sitter symmetry. For the black string, it has been shown explicitly that this value of q^2 is the dividing line between stable and unstable perturbations: smaller q^2 are unstable while larger q^2 are stable [29]. It is likely that the same will be true for the "extended bubbles". To avoid this instability, one must assume the extra dimensions are compactified and have size smaller than r_0, so that every $q^2 \neq 0$ is bigger than λ^2.

2.4 Quantum instability

Since the χ circle at infinity is contractible, there is only one choice of spin structure for fermions.[6] This corresponds to fermions which are antiperiodic around the circle, and hence supersymmetry is broken even asymptotically far away from the bubble by the Scherk-Schwarz mechanism. In string theory this type of supersymmetry breaking was studied by Rohm [11], and the same analysis applies to our backgrounds at large r. For large enough r_0, there are no tachyons from the string winding sector at infinity. However, at 1-loop order in string perturbation theory, a tadpole develops for the radius of χ, driving it to smaller values, toward the tachyonic regime (in analogy to the case of a non-supersymmetric compactification of M-theory [31]). The case of string theory corresponds to $D = 10$, and the 1-loop induced energy density there scales as $-1/r_0^{10}$ [11]. In addition, there is a nonperturbative quantum instability corresponding to the nucleation of additional bubbles far from the one we are studying.[7] The calculation of the rate for this process is identical to the original calculation of the decay of the Kaluza-Klein vacuum [9]. It is easy to see that the perturbative instability dominates over the nonperturbative instability. These instabilities mean that the background (2.3) does not provide a useful controlled time-dependent background (beyond the classical theory). However, there are generalizations which do, to which we now turn.

[6] In the case of $D = 4$ we are choosing the ordinary (trivial) spin structure for the ϕ circle.

[7] In the case of M-theory it has been argued that this is closely related (in the regime of small r_0) to the uncharged closed string tachyon instability [31].

3. Kerr bubbles

One can perform a similar double analytic continuation of rotating (Kerr) black holes, as discussed in [7]. Here we will study the dynamics of these solutions, uncovering some important new features. In the case where all angular momenta are nonzero, we argue that this construction leads to quantum-mechanically stable time-dependent backgrounds, assuming there are no classical tachyons from metric perturbations.[8] In the case where some (but not all) the angular momentum parameters are nonzero, we show that the solutions contain horizons for certain (but not all) observers. While these latter solutions do not appear to have perturbative quantum instabilities, they may still have nonperturbative instabilities associated with the nucleation of additional bubbles.

3.1 Four dimensional Kerr bubble

Let us start with the simplest case of the four-dimensional Kerr black hole. If one takes the Kerr metric in Boyer-Linquist coordinates and performs the double analytic continuation (2.2) together with an analytic continuation of the angular momentum parameter $a \to i\beta$, one obtains the metric

$$\begin{aligned} ds^2 \,=\, & -(r^2+\beta^2\sinh^2\tau)d\tau^2 + d\chi^2 + (r^2-\beta^2)\cosh^2\tau d\phi^2 - \\ & - \frac{r_0 r}{r^2+\beta^2\sinh^2\tau}(d\chi+\beta\cosh^2\tau d\phi)^2 + \frac{r^2+\beta^2\sinh^2\tau}{r^2-\beta^2-r_0 r}dr^2 \,. \end{aligned} \tag{3.1}$$

The radial coordinate is restricted to $r \geq r_b$ where r_b is the larger solution to the equation $r^2-\beta^2-r_0 r=0$. This minimal radius again describes a time-dependent bubble. In this case, one finds that regularity at the bubble requires that we make identifications[9] by the following symmetry operator

$$\hat{O} = e^{2\pi R\partial_\chi} e^{2\pi R\Omega\partial_\phi}(-1)^F \,, \tag{3.2}$$

where $R = 2r_0 r_b/\sqrt{r_0^2+4\beta^2}$ is the inverse of the surface gravity of the Kerr black hole (after the Wick rotation), $\Omega = \beta/r_0 r_b$, and F is the spacetime fermion number. This means that we need to have coordinate identifications

$$(\chi,\phi) \equiv (\chi+2\pi n_1 R, \phi+2\pi n_1 R\Omega + 2\pi n_2)\,, \qquad n_1, n_2 \in \mathbb{Z}\,. \tag{3.3}$$

At a fixed time, this metric describes a "bubble of nothing" excising a region near the origin of the orbifold of flat space

$$\frac{(\mathbb{C}\times\mathbb{R})}{\Gamma}\,, \tag{3.4}$$

where Γ is the group of identifications generated by (3.2) (where χ parameterizes the $\mathbb{R}$ factor and ϕ the angular direction in the complex plane $\mathbb{C}$ as in figure 3).

[8] The latter we are not able to check directly; this is related to the fact that this computation has so far proven to be prohibitively difficult also for higher-dimensional Kerr black holes, which are also not known for sure to be stable.

[9] For the ϕ circle we will choose a trivial spin structure and period 2π, although there are other possibilities. One could go to the universal covering space to make ϕ non-compact and then, if desired, one could compactify it on a circle of any period and spin structure.

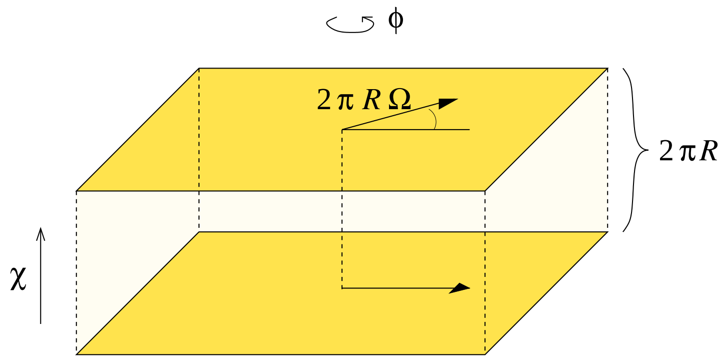

Figure 3: The orbifold of flat space corresponding to (3.3). The identification of the two darker planes in the figure involves a $2\pi R\Omega$ rotation of ϕ.

For rational values of $R\Omega$, the identifications (3.3) produce a "twisted circle" compactification of the sort recently studied in [12]. Note that this Kerr bubble is not rotating. There is no $d\tau d\phi$ term in the metric. The parameter β just describes the spatial identification.

An important difference between the identification (3.3) and the simpler identification of the Schwarzschild bubble is that now the compact dimension opens up asymptotically. In other words, the size of the compact circle grows with r. This has an important implication for supersymmetry, namely, if we consider (3.1) as a solution to supergravity then the space is asymptotically *locally* supersymmetric (in the sense of the SUSY breaking effects being suppressed at large r). It is only locally supersymmetric at infinity because a spinor transported all the way around the circle defined by (3.2) does not come back to itself, but the size of this circle goes to infinity as $r \to \infty$.

Let us look at the evolution of this bubble. For this it is useful to study the induced metric on the bubble $r = r_b$. The coordinate $\tilde{\phi} = \phi - \Omega\chi$ does not change under the identification (3.3), so it is a natural coordinate on the bubble, and the induced metric is given by

$$ds_b^2 = -(r_0 r_b + \beta^2 \cosh^2 \tau) d\tau^2 + \frac{r_0^2 r_b^2 \cosh^2 \tau}{r_0 r_b + \beta^2 \cosh^2 \tau} d\tilde{\phi}^2 \,. \tag{3.5}$$

For $\beta^2 \cosh^2 \tau \ll r_0 r_b$, the bubble evolves like de Sitter space, as in the case of the Schwarzschild bubble. However, for $\tau \to \infty$ the bubble metric becomes

$$ds_b^2 \to -\beta^2 d(e^\tau)^2 + \frac{r_0^2 r_b^2}{\beta^2} d\tilde{\phi}^2 \,. \tag{3.6}$$

This is simply a circle staying at a fixed proper radius: the bubble has stopped expanding! Similarly, for $\tau \to -\infty$, the bubble stays at a fixed size, and only begins contracting appreciably when $\beta^2 \cosh^2 \tau$ is comparable to $r_0 r_b$.

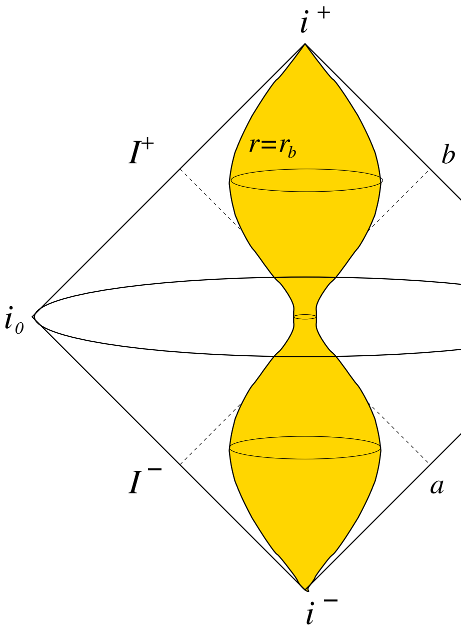

Figure 4: Penrose diagram of the four dimensional Kerr bubble, with null rays a and b separating the "Milne" and "de Sitter" epochs.

The behavior of the full metric at late times can be most easily seen by changing variables to $t = \sinh\tau$. The metric is then

$$\begin{aligned} ds^2 = &-\frac{r^2+\beta^2t^2}{1+t^2}dt^2 + d\chi^2 + (r^2-\beta^2)(1+t^2)d\phi^2 - \\ &- \frac{r_0 r}{r^2+\beta^2 t^2}(d\chi + \beta(1+t^2)d\phi)^2 + \frac{r^2+\beta^2t^2}{r^2-\beta^2-r_0r}dr^2 \,. \end{aligned} \tag{3.7}$$

At late times we see from this metric that the space around the bubble continues to expand, but with a local scale factor linear in the proper time (similarly to the Milne universe) rather than exponential. The acceleration of the de Sitter-like phase has stopped, and we enter a phase of mild time-dependence which we will refer to as the "Milne" epoch.

The Penrose diagram is shown in figure 4, with the asymptotic region equivalent to Minkowski space, in particular with a complete Minkowski null infinity. It is clear that this solution has no horizons. However, it is extremely well suited to studying the phenomenon of particle creation: it is an everywhere smooth weakly coupled background with mild time dependence in the past and future (and Minkowski null infinity), interrupted by a phase (the de Sitter phase) of strong time dependence. We will compute particle creation for some modes in this background in section 4, and interpret particle creation in string theory in section 5.

Since the Kerr bubble behaves qualitatively differently from the Schwarzschild bubble, it is natural to wonder if this is related to another qualitative difference between the spacetimes. Namely, in Kerr, the compact dimension opens up asymptotically, while in Schwarzschild it does not. We can check this by going to higher dimensions where there are solutions in which the compact dimension opens up in some directions and remains finite in others. We will find that in all cases, the bubble continues to accelerate in directions where the compact dimension approaches a constant radius, but stops accelerating in directions where the compact dimension opens up. It is as if the bubble "loses energy" when the dimension opens up.

3.2 Higher dimensional Kerr bubbles

We now turn to the dynamics of the higher-dimensional Kerr bubbles, which is our main case of interest. We start with the rotating black holes found by Myers and Perry [8]. The form of the metric differs in odd and even dimensions. We will consider the case of even spacetime dimension D. The metric has parameters associated with rotation in different orthogonal planes, so it is convenient to introduce spatial coordinates based on $D/2-1$ orthogonal planes: (μ_j, ϕ_j) with $\mu_j \geq 0$ and $0 \leq \phi_j \leq 2\pi$. There is one remaining spatial direction with coordinate α. Rather than work with the separate radial variables μ_j and α, Myers and Perry introduce an overall radial variable r and impose the constraint

$$\sum_j \mu_j^2 + \alpha^2 = 1\,, \tag{3.8}$$

so that $-1 \leq \alpha \leq 1$ and $0 \leq \mu_j \leq 1$.

The even dimensional Kerr black hole is [8]

$$\begin{aligned} ds^2 \;=\; & -dt^2 + r^2 d\alpha^2 + \sum_j \left(r^2 + a_j^2\right)\left(d\mu_j^2 + \mu_j^2 d\phi_j^2\right) + \\ & + \frac{r_0^{D-3} r}{\tilde{\Pi}\tilde{F}} \left(dt + \sum_j a_j \mu_j^2 d\phi_j\right)^2 + \frac{\tilde{\Pi}\tilde{F}}{\tilde{\Pi} - r_0^{D-3} r} dr^2\,, \end{aligned} \tag{3.9}$$

where

$$\tilde{F}(r,\mu_j) = 1 - \sum_j \frac{a_j^2 \mu_j^2}{r^2 + a_j^2}\,, \qquad \tilde{\Pi}(r) = \prod_j \left(r^2 + a_j^2\right)\,. \tag{3.10}$$

To obtain the Kerr bubble, we perform the analytic continuation

$$t = i\chi\,, \qquad a_j = i\beta_j\,, \qquad \alpha = i \sinh\tau\,, \tag{3.11}$$

giving

$$\begin{aligned} ds^2 \;=\; & -r^2 \cosh^2\tau d\tau^2 + d\chi^2 + \sum_j \left(r^2 - \beta_j^2\right)\left(d\mu_j^2 + \mu_j^2 d\phi_j^2\right) - \\ & - \frac{r_0^{D-3} r}{\Pi F} \left(d\chi + \sum_j \beta_j \mu_j^2 d\phi_j\right)^2 + \frac{\Pi F}{\Pi - r_0^{D-3} r} dr^2\,, \end{aligned} \tag{3.12}$$

where now

$$F(r,\mu_j) = 1 + \sum_j \frac{\beta_j^2 \mu_j^2}{r^2 - \beta_j^2}\,, \qquad \Pi(r) = \prod_j \left(r^2 - \beta_j^2\right)\,. \tag{3.13}$$

In light of the constraint (3.8), μ_j are now time dependent. It is convenient to extract this time dependence by setting

$$\mu_j = \hat{x}_j \cosh \tau \,, \tag{3.14}$$

with $\sum_j \hat{x}_j^2 = 1$. The minimum value of r is r_b which is defined to be the largest solution to $\Pi - r_0^{D-3} r = 0$. We again refer to this as the bubble. Regularity on the bubble requires identifications similar to (3.3),

$$(\chi, \phi_j) = (\chi + 2\pi R n_0, \phi_j + 2\pi R \Omega_j n_0 + 2\pi n_j) \,, \tag{3.15}$$

where

$$\Omega_j = \frac{\beta_j}{r_b^2 - \beta_j^2} \quad \text{and} \quad R = \frac{2 r_0^{D-3} r_b}{\frac{\partial \Pi}{\partial r}|_{r=r_b} - r_0^{D-3}} \,, \tag{3.16}$$

again with antiperiodic fermions for the n_0 identification.

We now describe the basic features of the Kerr bubble. First, one can check that for $D = 4$, this solution reduces to (3.1). In general, the time-time component of the metric is

$$g_{\tau\tau} = - \left[r^2 + \sum_j (\beta_j \hat{x}_j)^2 \; \sinh^2 \tau \right] . \tag{3.17}$$

We see the same behavior as in four dimensions. For small τ, the proper time along curves of constant r and $\hat{x}_j$ is proportional to τ, while for large τ, the proper time is proportional to e^τ. The only exception is if some of the β_j vanish. Then, in the directions orthogonal to all the planes of rotation, the second term in (3.17) vanishes and the proper time remains proportional to τ even at late time. It is clear that the metric components grow no faster than $e^{2\tau}$ at late time. Thus, generically distances expand linearly with proper time and the spacetime is again in a Milne phase at late time. However, in directions perpendicular to the planes of rotation, the expansion remains exponential for all time.

In four dimensions we saw that the bubble stops expanding completely at late time. This is not true in higher dimensions. One can see this by looking at the component of the metric in the ϕ_j direction. On the bubble ($\Pi = r_0^{D-3} r_b$), this is

$$g_{\phi_j \phi_j} = \frac{(r_b^2 - \beta_j^2) \hat{x}_j^2 \cosh^2 \tau}{F} \left[1 + \sum_{k \neq j} \frac{\beta_k^2 \hat{x}_k^2 \cosh^2 \tau}{r_b^2 - \beta_k^2} \right] . \tag{3.18}$$

The coefficient in front of the brackets is independent of τ at late time. In $D = 4$, there is only one rotation plane, so the sum inside the bracket is absent. Hence $g_{\phi_j \phi_j}$ approaches a constant and since the bubble is just this circle, the bubble stops expanding. In higher dimensions, $g_{\phi_j \phi_j} \sim e^{2\tau}$, so it generically expands exponentially with proper time initially, and later slows down and expands only linearly. However if some of the rotation parameters are zero, one can set $\hat{x}_j = 0$ for every nonzero β_j. This corresponds to looking in a direction orthogonal to all the planes of rotation. In this case $F = 1$, and the sum in (3.18) vanishes. Hence $g_{\phi_j\phi_j} \sim e^{2\tau}$ at late time, and since τ is proportional to proper time, the bubble continues to expand exponentially in these directions. Since the bubble expands exponentially in some directions and only linearly in others, it becomes highly distorted.[10]

The significance of the continued exponential expansion (even if it occurs only in certain directions) is that observers in these directions will lose causal contact and there will be (observer dependent) horizons. To see this, we first show that the metric orthogonal to the planes of rotation is very similar to the Schwarzschild bubble discussed in the previous section. Setting $\hat{x}_j = 0$ for every nonzero β_j, we see that the $d\chi d\phi_j$ cross terms vanish. Since $F = 1$, the resulting metric takes the form

$$ds^2 = \left(1 - \frac{r_0^{D-3} r}{\Pi}\right) d\chi^2 + \left(1 - \frac{r_0^{D-3} r}{\Pi}\right)^{-1} dr^2 + r^2(-d\tau^2 + \cosh^2\tau d\Omega^2)\,. \tag{3.19}$$

This metric differs from (2.3) only in the radial dependence of the χ, r plane. The analysis of the horizons in section 2.2 carries over exactly with the redefinition $f(r) = 1-(r_0^{D-3}r/\Pi)$. This shows that observers in the space orthogonal to the planes of rotation cannot communicate by sending signals in this space.

To see if there are horizons, we must investigate the motion of light rays off this subspace. When one does this, one finds that at late times two observers can both send signals to a third observer living off this subspace, but the third observer cannot send signals back to them (and hence they cannot communicate with each other). This is illustrated by looking at the asymptotic form of the solution near null infinity. If we take $r \gg r_0, \beta_j$, and $\tau \gg 1$, the metric becomes

$$ds^2 = \left[r^2 + \frac{1}{4}e^{2\tau}\sum_j(\beta_j\hat{x}_j)^2\right]\left[-d\tau^2 + \frac{dr^2}{r^2}\right] + \frac{r^2e^{2\tau}}{4}d\Omega_{D-3}^2 + d\chi^2\,. \tag{3.20}$$

Introducing null coordinates $v = \tau + \ln r$ and $u = \tau - \ln r$, the metric becomes

$$-e^v\left[e^{-u} + \frac{1}{4}e^u\sum_j(\beta_j\hat{x}_j)^2\right]dudv + \frac{1}{4}e^{2v}d\Omega_{D-3}^2 + d\chi^2\,. \tag{3.21}$$

Let us compare this with the asymptotic structure of Minkowski spacetime. In null coordinates $V = t+r, U = t-r$ the flat metric is $ds^2 = -dUdV + \frac{1}{4}(V-U)^2d\Omega^2$. Near future null infinity, $V \gg U$, so the metric is just $ds^2 = -dUdV + \frac{1}{4}V^2d\Omega^2$. This is clearly very similar to (3.21). In fact we can set $V = e^v$. In general we can also define a new U coordinate so (3.21) is similar to Minkowski spacetime. However on the subspace orthogonal to the rotation planes, $U = -e^{-u}$ only takes values less than zero. Thus, in these directions, the generators of null infinity are incomplete. This also happens for the Schwarzschild bubbles, and reflects the fact that the exponentially expanding bubbles hit null infinity.

[10] This phenomenon may also occur in gravity duals to gauge theories in metastable vacua [32].

Now if two observers hit different points on future null infinity, then they eventually lose causal contact with each other, although they can both send signals to a third observer in the interior. Since there are complete timelike geodesics whose past does not include the entire spacetime, these hybrid bubble spacetimes have horizons.

One might argue that these horizons are only present for a set of observers of measure zero, and hence are not of much physical interest. However, we now present a general argument that observers near the exponentially expanding part of the bubble are attracted to it. So it is likely that an open set of observers have horizons. Given any congruence of timelike geodesics with tangent vectors ξ^μ, the change in the convergence $c = -\nabla_\mu \xi^\mu$ along the geodesic satisfies the Raychaudhuri equation [33] which implies:

$$\dot{c} \geq \frac{c^2}{3} + R_{\mu\nu}\xi^\mu\xi^\nu \,. \tag{3.22}$$

If $R_{\mu\nu}\xi^\mu\xi^\nu \geq 0$ (which is equivalent to a condition on the stress energy tensor known as the strong energy condition) then c always increases along the geodesics. This just reflects the attractive nature of gravity. In de Sitter space, the timelike geodesics orthogonal to surfaces of constant global time are diverging at a constant rate, i.e., c is constant and negative. This is consistent with (3.22) since the last term on the right is negative: a positive cosmological constant does not satisfy the strong energy condition. However the bubble metrics we are considering are vacuum solutions, so the last term vanishes. Since timelike geodesics in part of the bubble are expanding as in de Sitter space, the only way (3.22) can be satisfied is if the nearby geodesics in other directions are converging toward them. Thus, nearby observers are attracted to the exponentially expanding part of the bubble. This clearly applies to the Schwarzschild bubble as well.

The higher dimensional Kerr metric with one angular momentum parameter nonzero can be dimensionally reduced along a circle to obtain a spherical brane expanding in a fluxtube [34]. It was previously shown that the brane continues to accelerate outward. This is consistent with our analysis since the brane lies on the higher dimensional bubble, and is expanding in directions orthogonal to the plane of rotation. Hence it does continue to accelerate. The fact that the rest of the bubble stops accelerating has apparently not been noticed previously.

3.3 Quantum stability

The quantum stress energy that is generated in these higher-dimensional Kerr backgrounds is expected to fall like $-1/r^{10}$ (in ten dimensions) since the radius of the compact direction grows asymptotically like r. Thus the equations of motion are unaltered at infinity (unlike the case of the Schwarzschild bubble, where one has a constant energy density of order $-1/r_0^{10}$ in the asymptotic region). As long as there are no infrared problems (i.e. as long as we rotate at least two planes), we expect that one can absorb the effects of the quantum-induced stress-energy by turning on small radial gradients of the dilaton and metric perturbations, supported near the origin of the rotated complex planes.

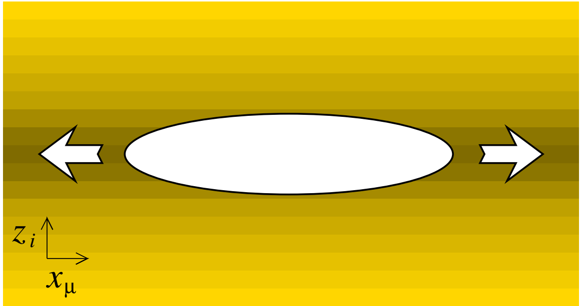

Figure 5: Asymmetric expansion of a hybrid bubble. In the darker region, local supersymmetry is broken to a larger extent; local supersymmetry emerges asymptotically at large z. It seems that the space-eating bubbles do not find the regions of unbroken local supersymmetry very tasty.

This also suffices to stabilize the orbifold (3.15) without a bubble present, for sufficiently large R so that there are no tachyons. One case of particular interest for the question of observables is the following. Consider the "twisted circle" orbifold

$$\frac{(S^1 \times \mathbb{C}^q)}{\mathbb{Z}_N} \times \mathbb{R}^{8-2q,1} \,. \tag{3.23}$$

Let us parameterize the complex planes by $z_1, \ldots, z_q$, the circle by χ, and the Minkowski factor by x^μ. For $q > 1$, the quantum stress-energy which would be generated near the origin of $\mathbb{C}^q$ in this background can be absorbed radially as just discussed. This produces a vacuum which has supersymmetry broken near the origin ($z_i = 0$) of the complex planes $\mathbb{C}^q$, with a small source of stress-energy localized near $z_i = 0$, but has asymptotic local supersymmetry for large z_i. By choosing appropriately the angular momentum parameters, one can embed a Kerr bubble into this spacetime (with rotation only in the $\mathbb{C}^q$ directions). It expands exponentially along the $\mathbb{R}^{8-2q}$ directions but stays near the origin of the complex planes $\mathbb{C}^q$ (see figure 5). The eternal acceleration of this background in the x^μ directions will be interesting for the discussion of observables below.

For Kerr bubbles with all angular momentum parameters turned on, the bubble stops accelerating as discussed above, and there are no horizons. It is tempting to conclude from this that the stabilization of the bubble geometry (which motivated our study of the Kerr bubbles as opposed to the Schwarzschild bubbles) removes the horizons, corroborating the point of view in [1].

However, the hybrid cases discussed above with eternal acceleration in a subset of the directions, provide one set of apparently perturbatively stable backgrounds without a standard S-matrix. Observers on the bubble separate exponentially in the x^μ directions, and lose causal contact with each other. They can send information off the bubble to a third observer, but they cannot send information to each other and no single observer can accumulate all the data in the full S-matrix.

We now consider possible nonperturbative quantum instabilities. Nonperturbatively, the backgrounds with all angular momentum parameters nonzero, are probably also stable against nucleation of additional bubbles. Since the compact direction opens up asymptotically, far from the first bubble the spacetime does not have the right boundary conditions

JHEP07(2002)007

to nucleate another bubble. This is roughly because the spacetime is essentially flat space asymptotically, which is stable. However, in the hybrid cases with horizons, there are directions where the compact direction remains finite, and the spacetime resembles the Schwarzschild bubble. In this case, there appears to be no reason why additional bubbles could not appear far from the first in these particular directions. If so, it would be interesting to investigate this further, studying what happens when the bubbles collide, and checking whether any horizons remain.

4. Particle creation in quantum field theory

A basic phenomenon which arises in generic time-dependent backgrounds is particle creation. This is particularly simple to study in backgrounds like the bubbles we described above with all angular momentum parameters nonzero, which have a Minkowski-like null infinity region where we can unambiguously define in and out vacua for massless fields. Because there is no global time translation symmetry, an initial positive frequency mode generally turns into a linear combination of positive and negative frequency modes in the future (and vice versa). In the Kerr bubble solutions, we have phases of mild time dependence asymptotically in the past and future (the "Milne" epochs), interrupted by the "de Sitter" epoch of exponential contraction and expansion of the bubble. Intuitively, we expect that at least for a long enough de Sitter phase, we should find the particle creation dominated by this epoch; this is confirmed in our analysis below.

Here we will study this process in detail for a massless scalar field[11] ϕ in the four-dimensional Kerr bubble geometry (3.1); we expect it to work similarly in higher dimensions with generic rotation angles, where this analysis would apply, for instance, to the dilaton in string theory. Similar particle-creation will also arise for other fields (like the graviton) but we will not discuss it here. Our goal is to express the future Minkowski modes in terms of the past Minkowski modes and to calculate the Bogolubov coefficients relating them.

One way to do this is to explicitly solve for the wave-functions of the field, by solving equations like (2.8) and its generalization to other backgrounds, and comparing the positive-energy modes at past null infinity with the positive-energy modes at future null infinity. However, solving these equations in general seems to be too complicated. Thus, we will follow another strategy, following [35], which is to consider frequencies $\omega \gg 1/r_0$ and make a geometric optics approximation. In other words, for each mode we will use the fact that the phase is (approximately) constant along each geodesic.

Near future null infinity, the phase of a future Minkowski mode is simply proportional to the coordinate distance between two geodesics. Following these geodesics to the past in the full geometry, we will find the phase as a function of the Minkowski coordinates near past null infinity, giving us the Bogolubov transformation.

More specifically, consider an s-wave mode, or some other low angular momentum mode. Decomposing this mode into a spherical harmonic and a function independent of the angular variables, one can reduce the problem to two dimensions. If the angular

[11]Note that the choice of coupling of ϕ to the metric will not be important because the scalar curvature vanishes in our backgrounds.

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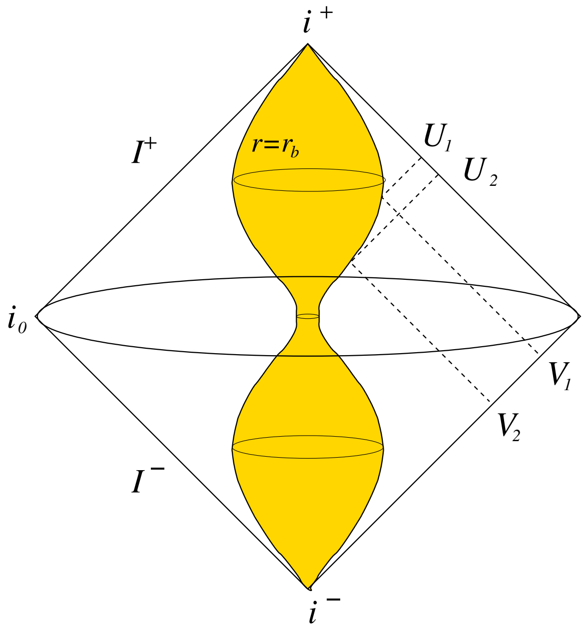

Figure 6: Pair of geodesics in the four dimensional Kerr bubble geometry used for the particle creation calculation.

momentum is small, it will not affect the effective 2d wave equation much and the mode will behave similarly to an s-wave. From now on we will express all quantities in the two-dimensional language.

We will be interested in an s-wave mode at $\mathcal{I}^+$ of purely positive frequency,

$$\phi \sim e^{i\omega U}\,, \tag{4.1}$$

with respect to a lightcone coordinate $U = T - X$, where X and T are the usual Minkowski coordinates in the radial and time directions at infinity. For this mode, the phase difference between two geodesics located at U_1 and U_2 will be $e^{i\omega\Delta U}$, where $\Delta U \equiv U_1 - U_2$ is the coordinate distance between the two geodesics (labelled 1 and 2 in figure 6) on $\mathcal{I}^+$. By solving the geodesic equation and tracing geodesics 1 and 2 back to $\mathcal{I}^-$, we can trace the phase difference between the values of ϕ on the two geodesics back to

$$e^{i\omega\Delta\,\mathrm{U}(\Delta V)}\,, \tag{4.2}$$

where again $\Delta V \equiv V_1 - V_2$ is the coordinate distance between the two geodesics on $\mathcal{I}^-$, and $V \equiv X + T$. This form reflects the fact that the mode has a constant phase (in the geometric optics approximation), but its expression in terms of U can be traded for that in terms of V on $\mathcal{I}^-$ by determining the distance between our two geodesics on $\mathcal{I}^-$.

For the four dimensional Kerr bubble (3.1), we can determine the null geodesics relevant for the s-waves by setting

$$ds^2 \equiv 0 \to d\tau^2 = \frac{dr^2}{r^2 - \beta^2 - r_0 r}\,. \tag{4.3}$$

Integrating this condition, we find the relation between r and τ satisfied along the null geodesics:

$$e^{\tau} = e^{-C}\left(r - \frac{r_0}{2} \pm \sqrt{r^2 - \beta^2 - r_0 r}\right), \tag{4.4}$$

where the integration constant C labels the geodesic, and where the $\pm$ comes from the two possible signs when taking the square root of (4.3), and distinguishes outgoing and incoming geodesics.

Since we are interested in the modes (4.1) at null infinity, we need to know the null coordinates U, V in terms of r, τ. They are given by

$$U = -re^{-\tau} + \frac{\beta^2}{4}\frac{e^{\tau}}{r}, \tag{4.5}$$

$$V = re^{\tau} - \frac{\beta^2}{4}\frac{e^{-\tau}}{r}. \tag{4.6}$$

In terms of U, V, the metric for large r ($r \gg r_b$, $r \gg \beta$, $r \gg r_0$) and large $|\tau|$ reduces to (for $d\chi = d\phi = 0$)

$$ds^2 = -dUdV. \tag{4.7}$$

Plugging (4.4) (with the plus sign) into (4.5) we find the future asymptotic U coordinate of the null geodesics as a function of C:

$$U \to \frac{1}{2}\left(-e^{C} + \beta^2 e^{-C}\right). \tag{4.8}$$

Similarly, the past asymptotic V coordinate of the geodesics is

$$V \to e^{-C}K_0 - \frac{\beta^2}{4}\frac{e^{C}}{K_0}, \tag{4.9}$$

where $K_0 \equiv \beta^2/2 + r_0^2/8$.

Let us fix the geodesic 1 in the figure to be a reference geodesic with a given value of C, $C = C_1$. We would like to solve for U_2 in terms of V_2. To start, let us solve for e^{C_2} in terms of V_2 using (4.9), and then plug the result into (4.8). We find

$$e^{C_2} = \frac{2K_0}{\beta^2}\left(-V_2 + \sqrt{V_2^2 + \beta^2}\right). \tag{4.10}$$

This leads to

$$\begin{aligned}\Delta U = U_1 - U_2 = &\frac{1}{2}\left(\frac{2K_0}{\beta^2}\left(-V_2 + \sqrt{V_2^2 + \beta^2}\right) - e^{C_1}\right) \\ &+ \frac{\beta^2}{2}\left(e^{-C_1} - \frac{1}{\frac{2K_0}{\beta^2}\left(-V_2 + \sqrt{V_2^2 + \beta^2}\right)}\right).\end{aligned} \tag{4.11}$$

It is worth remarking that in the limit of the Schwarzschild bubble, $\beta \to 0$, this reduces to

$$\Delta U^{(\beta=0)} = \frac{1}{2}\left(\frac{r_0^2}{8V_2} - e^{C_1}\right). \tag{4.12}$$

Plugging (4.11) into (4.2), we would like to project the result onto Fourier components $e^{\pm i\omega' V} \sim e^{\pm i\omega' V_2}$ (recall that we are keeping V_1 fixed), in order to read off the Bogolubov coefficients which determine the extent of particle creation in our background. From the nontrivial form of (4.11), we already see that our pure positive frequency mode in the future (4.1) does not continue back to a pure positive frequency mode in the past, and thus there is indeed particle creation.

More explicitly, from the standard definition of Bogolubov coefficients it follows that

$$\frac{1}{\sqrt{\omega}} e^{i\omega \Delta U} = \int d\omega' \left(\alpha^*_{\omega'\omega} \frac{1}{\sqrt{\omega'}} e^{-i\omega' V_2} - \beta_{\omega'\omega} \frac{1}{\sqrt{\omega'}} e^{i\omega' V_2} \right). \tag{4.13}$$

So, we can extract the Bogolubov coefficients $\alpha_{\omega'\omega}$, $\beta_{\omega'\omega}$ as

$$\alpha_{\omega'\omega} = \frac{1}{2\pi}\sqrt{\frac{\omega'}{\omega}} \int_{-\infty}^{\infty} dV e^{i\omega' V} e^{i\omega \Delta \, \mathrm{U}(V)}, \tag{4.14}$$

$$\beta_{\omega'\omega} = \frac{1}{2\pi}\sqrt{\frac{\omega'}{\omega}} \int_{-\infty}^{\infty} dV e^{-i\omega' V} e^{i\omega \Delta \, \mathrm{U}(V)}, \tag{4.15}$$

where $\Delta \, \mathrm{U}(V)$ is given explicitly by (4.11), and where we are ignoring overall phases since the quantities we are interested in, such as the number of particles produced, are determined by the magnitude of $\beta_{\omega'\omega}$. For a fixed mode ω, the total number of particles produced (in a range between ω and $\omega + d\omega$) is given by

$$dN_\omega = d\omega \int_{1/r_0}^{M_P} d\omega' |\beta_{\omega'\omega}|^2 , \tag{4.16}$$

where we have restricted the integral to our regime of validity, the lower end coming from the validity of the geometric optics approximation and the upper end from the inapplicability of quantum field theory techniques at the Planck scale.

From these exact results we would like to extract certain qualitative features of the physics. First, we would like to check that the particle creation is not large enough to back-react significantly on the geometry. Second, we would like to understand where (or when) the bulk of the effect arises, and to test our intuition that the particle creation is dominated by the de Sitter epoch in the Kerr bubble spacetime.

In order to accomplish this, let us consider first a stationary phase approximation to the integrals (4.15), (4.14). In the Schwarzschild bubble, we have $\beta = 0$ and thus $V \gg \beta$ for any nonzero V. Let us check whether the stationary phase point of the full solutions (4.15), (4.14) is in this regime. If so, then the de Sitter phase indeed dominates the particle creation. In the limit where $V \gg \beta$ and $\beta^2/r_0^2 \ll \omega/\omega'$ (the Schwarzschild bubble approximation, whose self-consistency condition we will discuss momentarily), the stationary phase is at

$$V^2 = V_s^2 \equiv \pm \frac{K_0}{2}\frac{\omega}{\omega'}\,, \tag{4.17}$$

for $\alpha_{\omega\prime\omega}$ and $\beta_{\omega\prime\omega}$ respectively (namely, the phase V_s is real for (4.14) and imaginary for (4.15); in the latter case we must deform the contour to go through the stationary phase point). In order for this approximation to be self-consistent, from (4.17) and the definition of K_0 above we need

$$\frac{\omega}{\omega'} \gg \frac{\beta^2}{r_0^2}\,. \tag{4.18}$$

The smallest ratio ω/ω' that we can discuss reliably is $1/(r_0 M_P)$. We can thus choose β small enough so that (4.17), (4.18) are satisfied for all the frequencies we consider. This regime of very small β is of interest in any case for producing a long de Sitter phase.

Plugging the stationary phase (4.17) into the integrands (4.14), (4.15), we arrive at the stationary phase approximation to the Bogolubov coefficients:

$$\alpha_{\omega'\omega} \sim e^{i\frac{r_0}{2}\sqrt{\omega'\omega}}\,, \tag{4.19}$$

$$\beta_{\omega'\omega} \sim e^{-\frac{r_0}{2}\sqrt{\omega'\omega}}\,. \tag{4.20}$$

In the Schwarzschild bubble limit (which, as just discussed, is a good approximation to the Kerr case as well for small enough β) one can evaluate the integrals (4.14), (4.15) exactly in terms of Bessel functions. The results are

$$|\alpha_{\omega'\omega}| = \frac{r_0}{8} H^{(1)}_{-1}\left(\frac{1}{2} r_0 \sqrt{\omega\omega'}\right) \tag{4.21}$$

(where H is a Hankel function) and

$$|\beta_{\omega'\omega}| = |\alpha_{\omega'(-\omega)}| = \frac{r_0}{4\pi} K_1\left(\frac{1}{2} r_0 \sqrt{\omega\omega'}\right) \tag{4.22}$$

(where K is a modified Bessel function); these have the same behavior as (4.20) and (4.19) in an asymptotic expansion.

From (4.20), (4.22) we see that the nontrivial Bogolubov coefficient $\beta_{\omega'\omega}$ dies to zero exponentially for large frequency, so that one expects[12] the produced particles to carry finite energy, in contrast to the situation in the models of [16] and the non-euclidean conformal vacua in [2], and similarly to the finite-energy spacelike branes in [17]. The particle creation in our backgrounds is soft at high frequency, and is therefore consistent with our QFT analysis. Because of this exponential suppression, the total number of particles produced, determined from (4.16) by integrating over ω (with the IR cutoff $1/r_0$), is finite. Since these particles can spread over the infinite region of null infinity, the energy density produced is vanishingly small and the particles we produced do not back-react significantly on the geometry.

It is interesting to note that the Bogolubov coefficients (4.21) and (4.22) are of exactly the same form as those obtained for a mirror moving with a constant proper acceleration (i.e. along a hyperbolic trajectory) in flat space [5]. So there is a close analogy to the previously studied moving mirror problem. However, whereas the moving mirror was an ad hoc construct, requiring an external force to keep the mirror accelerating, the bubble spacetimes are solutions to Einstein's equations.

[12]Note however that the relation between particles and the energy-momentum tensor is usually very complicated in a time-dependent spacetime.

JHEP07(2002)007

In summary, within the range of frequencies between $1/r_0$ and M_P, we find a rich spectrum of particles produced by the background, but not enough to significantly back-react on the geometry. In the next section we will discuss the string theoretic description of particle creation in general terms, which will apply in particular to our bubble geometries.

5. Particle creation in string theory and non-local string theories

Most of our analysis, including the computation of particle creation we just completed in section 4, has been in the framework of low-energy effective quantum field theory and general relativity. In this section we will move on to string theory. In section 5.1, we will present an observation concerning the description of particle creation in string theory, in particular showing that Nonlocal String Theory (NLST) [13] arises naturally on the worldsheet in describing the resulting squeezed states. This assumes that there is a generalization of string perturbation theory to time dependent backgrounds — perhaps in backgrounds such as our bubbles this generalization proceeds via translation from the euclidean continuation. Such a generalization has yet to be formulated however, and we discuss some of the challenges involved in doing so in section 5.2.

5.1 Particle creation in string theory

To describe gravitational particle creation from the string theory point of view, we will use general methods known from quantum field theory in curved space and translate them into perturbative string theory.

One way to formulate particle creation is to look (in the Heisenberg picture) at a vacuum state $|\mathrm{in}\rangle$ which does not have any particles in it; i.e. it is killed by the annihilation operators a_{in}, obtained from a mode expansion of the field which reduces in the far past to an ordinary Fock space expansion in which creation operators $a_{\mathrm{in}}^{\dagger}$ multiply pure positive frequency modes and annihilation operators a_{in} multiply pure negative frequency modes. In general, the modes multiplying $a_{\mathrm{in}}^{\dagger}$ and a_{in} do not reduce in the far future to pure positive frequency and pure negative frequency modes, respectively. That is, the state $|\mathrm{in}\rangle$ is not the same as the state $|\mathrm{out}\rangle$ killed by the operators a_{out} multiplying the pure negative frequency modes of the fields in the future. Instead, it is a squeezed state $|\Psi_\kappa\rangle$, which can be written in terms of a basis of out-going oscillators ($a_{\mathrm{out}}^I|\mathrm{out}\rangle = 0$) as

$$|\Psi_\kappa\rangle \equiv |\mathrm{in}\rangle = Ce^{\kappa_{IJ}a_{\mathrm{out}}^{\dagger I}a_{\mathrm{out}}^{\dagger J}}|\mathrm{out}\rangle\,, \tag{5.1}$$

where C is a normalization constant (these states are normalizable for sufficiently small κ), and $\kappa = -\frac{1}{2}\beta\alpha^{-1}$ in terms of the matrices α, β of Bogolubov coefficients defined as in (4.13).

When we calculate S-matrix elements using the bra state $\langle\Psi_\kappa| = \langle\mathrm{in}|$ (and oscillator excitations above it) in the future, this amounts to treating the particles produced as part of the background.

Another way to phrase particle creation is by looking at correlation functions in the "empty" $|\text{in}\rangle$ and $|\text{out}\rangle$ vacua. The above discussion makes it clear that in general time-dependent backgrounds correlation functions of the form

$$\langle \text{out}| a_{\text{out},1} \cdots a_{\text{out},2n} \, |\text{in}\rangle \tag{5.2}$$

or

$$\langle \text{out}| a^\dagger_{\text{in},1} \cdots a^\dagger_{\text{in},2n} \, |\text{in}\rangle \tag{5.3}$$

will be non-vanishing, and will correspond to the probability for finding $2n$ outgoing particles when starting from the initial vacuum state (or vice versa). On the other hand, the correlation function similar to (5.3) with $\langle \text{in}| = \langle \Psi_\kappa|$ appearing instead of $\langle \text{out}|$ will vanish for $n > 0$. In our discussion we will be assuming that one can construct asymptotic scattering states for the background of interest. We are also assuming that interactions can be neglected, so that the free field result for the particle creation is a good approximation, as in section 4 where the coupling can be made arbitrarily small and the particles created have a vanishing number *density*. In general, the fields in the Heisenberg representation, or the states in the Schrödinger representation, will evolve nontrivially and leave us with something more complicated than the simple pure squeezed state in (5.1). In situations with an ambient temperature, the interactions would be expected to lead to thermalization, and in general one expects decoherence to occur effectively for observers not privy to the global structure of the state.

In string theory, we expect that as in flat space, the creation and annihilation operators for the asymptotic scattering states will be related to vertex operators $V_\pm^{\text{in,out}}$, where the sign depends on the sign of the energy with respect to the corresponding vacuum. We expect in cases which have an S-matrix (such as our Kerr bubbles with all rotation parameters non-zero), that just as in flat space, S-matrix elements between ket states created by the $a^\dagger_{\text{in}}$'s on the $|\text{in}\rangle$ vacuum and bra states created by a_{out}'s on the $\langle \text{out}|$ vacuum correspond to worldsheet correlation functions of the corresponding integrated $V_+^{\text{in}}(z)$ and $V_-^{\text{out}}(z)$ vertex operators. Just as in field theory, the in and out bases are not independent, and we can express the in vertex operators as linear combinations (Bogolubov transformations) of out operators and vice versa.

The correlation functions we compute in field theory depend on the initial and final states we use, so the results we expect to find in string theory depend on which initial and final states we have there. If we start from the euclidean string theory (in cases for which such euclidean versions exist), the analytic continuation defines a particular state which we can call the "euclidean vacuum" $|\text{euclidean}\rangle$. Generally, this is not the same as the natural lorentzian vacua with no particles (see, e.g., [36, 2]), but rather it looks like a squeezed state. It seems that correlation functions in euclidean string theory will naturally continue to lorentzian correlation functions in this particular state. It is interesting to ask how we can compute in string theory correlation functions in the usual empty lorentzian vacua $|\text{in}\rangle$ and $|\text{out}\rangle$, or in particular interesting squeezed states like $\langle \Psi_\kappa|$. We will argue that to do this we need to deform the worldsheet action by non-local terms, as in [13].

In order to see this, let us first recall how to describe different initial and final states in a path-integral formulation of field theory. If (staying in the Heisenberg representation so that the fields evolve but the states do not) we consider states which are eigenstates of the field $\phi(T_i)$ in the asymptotic past and $\phi(T_f)$ in the asymptotic future (for string theory we will be interested in the limit $T_{f,i} \to \pm\infty$), we have the relation

$$\langle \phi(T_f) = \phi_b | \phi(T_i) = \phi_a \rangle = \int [d\phi] \Big|_{\{\phi(t=T_i)=\phi_a , \phi(t=T_f)=\phi_b\}} e^{iS} \,, \tag{5.4}$$

so the boundary conditions in the path integral language correspond to the choice of states appearing in the matrix element in operator language. If we want to consider more general wavefunctions $|\Psi_{a,b}\rangle$ instead of $|\phi_{a,b}\rangle$, then we simply decompose them in the $|\phi\rangle$ basis :

$$|\Psi\rangle = \int [d\phi] |\phi\rangle \langle \phi \Psi\rangle \,, \tag{5.5}$$

giving

$$\langle \Psi_b | \Psi_a \rangle = \int [d\phi_a] \int [d\phi_b] \int [d\phi] \Big|_{\{\phi(T_i)=\phi_a , \phi(T_f)=\phi_b\}} \Psi_b^\dagger[\phi(T_f)] \Psi_a[\phi(T_i)] e^{iS} \,. \tag{5.6}$$

We are interested in particular in wavefunctions $|\Psi\rangle$ which are squeezed states. Above we wrote such squeezed states in terms of creation operators (5.1), but we can equivalently write them in terms of the fields as gaussian wavefunctions of the form

$$\Psi_s \sim C e^{\tilde{c} \int d^{d-1}\sigma d^{d-1}\sigma' \phi_+(\sigma) \Delta(\sigma,\sigma') \phi_+(\sigma')} \,, \tag{5.7}$$

where $\tilde{c}$ is a constant, σ is a coordinate on the boundary ∂ in terms of which we define the squeezed state (this boundary can be space-like or null), and $\phi_\pm$ are the positive and negative frequency parts of the field at that boundary. We have taken into account the fact that in general the wavefunction will not be local on ∂ (see, e.g., [2]).

To describe matrix elements between these states, we can plug them into (5.6). Since the wavefunctions (5.7) inserted in the path integral in this case are exponentials of the form $\Psi_s = e^{W[\phi]}$, we can reinterpret the log of these insertions as relatively simple contributions to a boundary action. In general these actions will not be local on the boundary, as expected since squeezed states embody long-range correlations, but in some special cases the boundary action may be local.

Having obtained the effect of the squeezed states as a shift in the boundary action, we can now deduce the string theory description. We can consider the boundary action we just derived as part of the interaction hamiltonian, and treat it perturbatively in the parameter $\tilde{c}$ appearing there. Bringing down powers of the boundary action into correlation functions leads to contributions to amplitudes in which those boundary fields are contracted with fields in other vertices in the diagram. This has the same effect as adding external (integrated) vertex operators in string theory. Thus, the squeezed state of a space-time field ϕ is reproduced in string theory by introducing a shift in the worldsheet action by the corresponding multilocal function of integrated vertex operators corresponding to ϕ. Note that we avoided issues of operator ordering by writing the squeezed state purely in terms of positive frequency modes.

For instance, in the case of (5.1), a correlation function involving $\langle \Psi_\kappa |$ would be described by

$$\delta S_{ws} = \sum_{I,J} \left(-\frac{1}{2}\beta\alpha^{-1} \right)_{IJ} \int d^2 z_1 V_+^{\mathrm{out},I}(z_1) \int d^2 z_2 V_+^{\mathrm{out},J}(z_2) + \delta S_{E,\mathrm{in}} + \delta S_{E,\mathrm{out}} \tag{5.8}$$

which is manifestly non-local on the worldsheet.[13] Here we have included terms $\delta S_{E,\mathrm{in}} + \delta S_{E,\mathrm{out}}$ describing the (nonlocal) shift from the euclidean vacuum to the empty lorentzian vacua, which are applicable if we formulate the string theory using the euclidean continuation. The appearance of worldsheet non-locality here (and its connection to a boundary action) is very similar to its appearance in the discussion of multi-trace operators, related to multi-particle states in AdS, in [13] (and its interpretation in supergravity on AdS in terms of a boundary action [37]). In general the sum in (5.8) will be replaced by an integral over all possible outgoing modes (if we are describing the final state, or over incoming modes if we are describing the initial state). The form of the deformation above is valid when its coefficient is very small, in which case the deformation manifestly preserves worldsheet conformal invariance; for a finite coefficient the form of the deformation would generally receive corrections, corresponding to the backreaction of the state on the background. Our discussion here was for the case of a single free scalar field (such as the dilaton), but there is no problem (in principle) in generalizing it to other cases.

Using this formalism, we can translate a given initial or final squeezed state into a non-local contribution to the worldsheet action. Given a string theory for the backgrounds we discussed above, we can explicitly write down the deformation we would need for describing the natural final squeezed state there (assuming that our original string theory described the "empty" vacuum state). For example, at leading order in α', the string worldsheet action in our bubble backgrounds shifts to include a term of the form (5.8), with $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ as given in section 4, and with the massless vertex operators at leading order given by the solutions of the wave equation (such as (2.8)) for the corresponding spacetime fields. We expect that at low energies the string theory results derived from these actions will reproduce field theory correlation functions in the appropriate initial and final states.

As another example, if we have a string theory describing a background including dS_3, and we want to do computations in the final squeezed state (which is the time-evolved initial vacuum state for a particular scalar field on dS_3), then using the results of [2], we need a similar nonlocal deformation of the worldsheet action. Interestingly, the dS/CFT conjecture would relate these deformations to double-trace deformations in a dual CFT, which is similar to the way the NLST deformations were discovered in the context of double-trace deformations of AdS/CFT [13]. One should, however, keep in mind that for these squeezed states there is a large backreaction on the geometry. Also, it appears hard to define the gravity counterpart of the exact CFT correlators, taking into account non-perturbative aspects of gravity in de Sitter space [26].

[13] The action is not real, but this should not be surprising when we have particle production and we are looking at a complex final state.

In the AdS/CFT context, it was shown in [37] that one could simply describe the NLST deformation in a formal supergravity approximation as a deformation of the boundary action for the fields in AdS.[14] This boundary action, on the timelike boundary of AdS, implements deformed boundary conditions for the supergravity fields. In our case here, we have a boundary action corresponding to the squeezed state we consider in the future, on null or spacelike infinity. Both types of boundaries lead to an NLST deformation on the worldsheet.

Thus, NLSTs arise naturally whenever we want to compute correlation functions in states which are not the natural initial and final states of string theory (e.g. the states defined by the euclidean continuation). This does not mean that we *have* to use NLST for time dependent backgrounds; if we calculate matrix elements (or equivalently the corresponding path integral) involving the "euclidean vacuum" state $\langle\text{euclidean}|$ instead of the squeezed state $\langle\Psi_s|$ we would use ordinary "local" string theory rather than NLST. However, in many contexts it may be natural to use a final state squeezed with respect to $\langle\text{euclidean}|$ (at least before taking into account the effects of decoherence), and then NLST seems to be required. Generally NLST seems to be required even to describe the natural lorentzian vacua, if we use a continuation from euclidean space to formulate string perturbation theory, since in time-dependent backgrounds the lorentzian vacua are related by a non-trivial Bogolubov transformation to the "euclidean vacuum". The S-matrix is presumably related to correlation functions of the vertex operators $V_\pm^{\text{in,out}}$ in the lorentzian vacua $\langle\text{out}|$ and $|\text{in}\rangle$. If we can formulate string theory directly in lorentzian space, then the local string theory may correspond to doing computations in the natural lorentzian in and out vacua (though this is not obvious), but we would need NLST to do computations directly in squeezed states, such as the ones arising by particle creation from the vacuum. Note that the formulation we described in this section applies also to Minkowski space, so we could choose also there non-trivial (correlated) initial and/or final states, which would be described by a non-local theory on the worldsheet. We see that worldsheet non-locality can arise in much more general contexts than anti-de Sitter space where it was first discussed [13]; it would be interesting to understand more generally when such non-locality occurs, and how to quantize and do computations in these non-local theories (beyond perturbation theory in the non-local deformation).

In our discussion here we generally ignored the backreaction of the initial or final state on our background. Obviously, when squeezed states with many particles are involved, this backreaction may not be negligible and needs to be taken into account. This may change our conclusion that any choice of state is allowed, and is realized by a particular non-local deformation of the worldsheet action. As we saw in section 4, in our bubble geometries the back-reaction is small.

A somewhat confusing aspect of the discussion here is that even if we compute with the local worldsheet theory, it seems that 2-point functions of incoming or outgoing vertex operators on the sphere should be non-zero. Usually in string theory, 2-point functions

[14]It is worth emphasizing that this boundary action is local in the AdS coordinates but it is nonlocal on the compact part of space, and that the worldsheet description is nonlocal.

vanish on the sphere because of the infinite volume of the group of conformal transformations preserving two points, except when there is another infinity to cancel this (as in for example [38]). In our case it is not clear where such an additional infinity would come from, though conceivably it could come from integrating over the time direction.

5.2 Time-dependent backgrounds in string theory

In general, the description of time-dependent backgrounds (without any light-like Killing vector) in string theory is a notoriously difficult subject. (String theory on plane wave backgrounds, which are time-dependent but independent of a light-cone time direction, was studied in many papers starting with [39].) One longstanding problem [40], which is starting to receive more attention [41], is that most known time-dependent backgrounds, and in particular the ones which are relevant for cosmology, have singularities. Such singularities pose an obvious problem for low-energy gravity computations, but it may be that these are resolved in string theory either classically or quantum mechanically and pose less of a problem there.

A complementary direction is to avoid the singularities by considering for example de Sitter space or the bubble solutions we have studied in this paper, which allow one to focus on the many other important questions in string cosmology. Even the study of non-singular time-dependent backgrounds is problematic in string theory. Among the unsolved problems are how to deal with cosmological horizons, how to prove a no-ghost theorem, and how to define physically meaningful observables. An additional problem is that string perturbation theory as we know it currently is only well-defined either in light cone gauge (which is not available in generic time-dependent backgrounds) or in euclidean space — both on the worldsheet and in spacetime — where one has the usual genus expansion.

The simplest way to try to define lorentzian string theory may be by analytic continuation of results from euclidean space. For example, the bubble geometries we study here, which are double analytic continuations of black hole solutions, have euclidean continuations which have been well studied in the black hole context [30]. However, some time-dependent backgrounds do not have a straightforward euclidean continuation, so it is not clear how to even formulate string perturbation theory in such backgrounds. Furthermore, even in cases such as ours which do have a euclidean continuation, there are issues involved in translating euclidean computations to lorentzian ones, as we will see shortly.

One of the first issues we need to address when formulating string theory on a bubble background of the type discussed above, is the question of whether the leading order solution in the α' expansion extends to a full solution of classical string theory. Unlike some other time-dependent backgrounds like lorentzian orbifolds and the Nappi-Witten background [42], our backgrounds are not exact CFTs. However, they are solutions to Einstein's equations so they are conformal at 1-loop order, and there are no singularities or regions of strong coupling so both the α' expansion and the string loop expansion are good everywhere in the spacetime. As we discussed in section 3, the instabilities introduced by the string loop expansion (tadpoles introduced by quantum mechanically generated stress energy) are localized in space in more than two directions, and thus can be absorbed by mild radial variations of the supergravity fields. The corrections to the field equations

JHEP07(2002)007

arising from α' effects will also be localized, and may be absorbable similarly in small radial variations of the supergravity fields. In particular, in analyzing massless scalar fields (and gravitons in the four-dimensional Schwarzschild case) in our study of classical stability in general relativity, we found no massless (or tachyonic) modes localized near the bubble. If this persists to hold for all massless fields in all examples, then the solutions are isolated, and α'-induced tadpoles will shift slightly but not destabilize the solution.

On the worldsheet, this question of classical stability amounts to the following. We solved the β-function equations to 1-loop order in α' by solving Einstein's equations. At higher loop orders we expect to find non-zero beta functions which will induce some flow, and the question is whether this flow has a fixed point which is close to the original background (with corrections of the order of the curvature in string units). It is natural to study this question in euclidean space, where we expect the worldsheet CFT to be unitary. The euclidean continuation of our backgrounds is the same as the euclidean continuation of the Schwarzschild and Kerr black hole backgrounds, so the existence of classical (euclidean) string theory in our backgrounds is equivalent to the existence of classical string theory on the black hole backgrounds. As in the bubble backgrounds, in the lorentzian black hole backgrounds we expect an isolated solution and no instabilities from α' that cannot be absorbed in small radially-dependent shifts of the fields [43].

There are, however, some difficulties with implementing this program. One immediate issue is that not all the vertex operators we would like to have in the lorentzian case exist in the euclidean case, since in the latter solution there are angular coordinates with a discrete spectrum that become noncompact (time) dimensions in the lorentzian continuations.

Another issue is the fact that the euclidean continuation has a negative mode [30] related to the negative specific heat of the corresponding black hole. This has two interesting consequences.

First, this mode translates into a relevant operator on the euclidean string worldsheet CFT, so that the euclidean black hole target space is an unstable fixed point with respect to variations of the corresponding coupling in the worldsheet sigma model. A generic RG flow would lead us far away from this fixed point. However, since the solution is isolated (there being no marginal deformations), the α' corrections will only shift the unstable fixed point slightly, and we can fine tune the worldsheet theory order by order in α' to remain at the unstable fixed point. This fixed point will describe a background only slightly shifted from the leading order in α' general relativistic solution we have been working with in the bulk of this paper.

Second, this negative mode leads to a divergent one-loop path integral (genus one amplitude) in the euclidean continuations, even though the corresponding black holes and bubble solutions do not have tachyonic instabilities (at least in $D = 4$). It may be possible to interpret this divergence as follows.[15] One may be able to analytically continue this divergent computation to obtain a finite answer, at the cost of introducing an imaginary part to the amplitude. In a theory with the ordinary unitarity relations, the imaginary part

[15]We have enjoyed useful discussions with T. Banks, M. Berkooz, D. Kutasov, and A. Lawrence on this issue.

of the one-loop amplitude would be equal to the square of the amplitude to produce pairs of particles. In time-dependent backgrounds, we do not know the appropriate generalization of the cutting rules, but since one can obtain unitary evolution formally for quantum fields on this space (with a time dependent hamiltonian) one may be able to make this argument precise including the effects of the time-dependence. If so, then the analytically continued one-loop vacuum amplitude could provide an alternate means to calculate the particle creation amplitude in the full string theory, generalizing the results of section 4.

As in the case of static backgrounds, developing explicit controlled solutions such as those we have studied here is an important prerequisite to addressing these very basic questions about formulating string theory in time-dependent backgrounds. With such a formulation in hand, we could answer interesting questions about how UV-sensitive quantities behave in time-dependent string backgrounds (for example the question of whether string theory softens particle creation effects at high energies), and perhaps we would get a better handle on classical singularities and on more realistic backgrounds.

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JHEP07(2002)007